AN ECONOPHYSICS FRAMEWORK TO UNDERSTANDING COMPLEX FINANCIAL PHENOMENA: FOREIGN VOLATILITY TRANSMISSION.

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BSc. Economics (2023-26)

Abstract:

This research investigates the possibilities in using an econophysical model of differential geometry to track and measure the unidirectional transmission of foreign volatility between markets of different countries. The model framework may be helpful when extended to measure the transmission pathways (both uni- and multi-directional) of volatility across market systems.

By quantifying through econophysics the extent to which shocks in one market system seep into other market systems, the study aims to better capture cross-market volatility spillovers and their dynamic evolution over time.

Multiple econometric and volatility modeling approaches - including GARCH-based and time-varying correlation frameworks are weighed on its accuracy or comprehensibility to estimate the degree of imported volatility. This analysis contributes to understanding global financial interconnectedness and offers a robust comparison on methods to calculate volatility transmission mechanisms across markets.

Through the framework introduced, this study seeks to view through a new lens cross-market volatility linkages and provide empirical insights relevant for quantitative finance, systemic risk monitoring, and policy formulation in emerging financial markets as markets become more and more complex.

As discussed in this study, econophysics is a field of much potential, with some applications mentioned here, to help ease into the relatively newer way of looking at complex economic phenomena.

Introduction to Volatility Transmission:

While talk has been plenty at the academic table surrounding the increasing "noodle bowl" effect of financial markets abroad, these talks were fueled largely due to the damages accrued during the 2008 Global Financial Crisis.
Debate on methodology to better understand the transmission mechanism of volatility and risk beyond ripple effects within the domestic market have led to varied approaches to capturing the extent of affection of the globalized mechanism of "travelling risk" that leads to a country "importing" much of volatility originated beyond its borders.

By studying new ways on the topics of how and when volatility is transmitted across borders - we achieve 3 things:

1. Policymakers can anticipate contagion effects, design buffers to protect domestic markets, and calibrate monetary and regulatory responses more precisely. This knowledge transforms *reactive* policymaking into *proactive* risk management. As complex products in finance are developing - like REITS, and crypto, it is essential to understand and build systems and environments that are better capable to evolve with its contents within.

2. Investors are better informed of the behaviours of the market, leading to less mis-calculated speculation, if any speculation at all, and a stronger basis of sentiment leading to robust standing of the market, which ultimately results in a positive reinforcement loop.

3. In an ever changing financial environment, the only constant parameter is volatility. Irony of the matter being the dual nature of volatility - both latent and observable. To grasp within reach the driving forces behind the mathematical behaviour of volatility could open gateways in academic research to sophisticated models, and powerful predictors that would be especially helpful as we enter into the newly emerging era of Artificial Intelligence.

Providing a small timeline to outline advancements made in this area-

Literature and academic research work regarding volatility transmission surged post 1990's, with each text exploring different approaches to the problem. (Hamoa et al (1990), Kumar & Mukhopadhyay (2002), Tomiwa Sunday Adebayo et al (2024), Bae and Karolyi (1994), Karolyi, (1995), Singh et al (2010), Theodossiou and Lee(1993) etc.), of which broad concepts and takeaways will be discussed and perused in this section to stand on the shoulders of giants.
Hamoa et al (1990) [1]still stand as one of the widely cited academicians for their work in this field.

[1] https://finance.martinsewell.com/stylized-facts/volatility/HamaoMasulisNg1990.pdf

By the late 1990s, research on *"financial contagion"* further expanded the discussion. Researchers debated whether crises create new cross-market volatility linkages (contagion) or simply strengthen existing interdependence. Forbes and Rigobon (2002) famously argued that after correcting for heteroskedasticity, there was little increase in cross-market correlations during crises - implying no true "contagion" is present.[2]

Others disagreed, finding structural breaks in volatility transmission during turbulent periods. For example, an IMF study defines "shift contagion" as a significant change in transmission mechanisms during crises[3]. Overall, since the 1990s the literature evolved from simple correlation analysis to more sophisticated models capturing time-varying volatility linkages, as discussed next.

Pia Nandini Malaney's *"The Index Number Problem: A Differential Geometric Approach"*(1996) [4]reinterprets one of economics' classical measurement puzzles - how to construct a consistent price or quantity index - through the lens of differential geometry.

Malaney introduces an "economic derivative" that adapts the mathematical notion of constancy to reflect purchasing power rather than raw numerical equality. This geometric reformulation shows that the inconsistencies between various traditional index formulas (Laspeyres, Paasche, Fisher, etc.) arise not from algebraic flaws but from using the wrong type of derivative. By redefining constancy using a covariant derivative, she constructs a unique, internally consistent index that converges to the Divisia index, resolving a century-old problem in index theory.

Extrapolating this advancement and relatively new aspect on approaching the subject at hand, my objective is to try and quantify the transmission process into an econophysics model that uses differential geometry to track inter-market phenomena such as cross-market volatility transmission.

But why attempt a differential geometric approach for the same? This question binds with the existing research gap that has persisted due a number of factors that will be discussed in the following section.

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[2] https://www.nber.org/system/files/working_papers/w7267/w7267.pdf

[3] ecb.europa.euecb.europa.eu

[4] Malaney, Pia. "The Index Number Problem: A Differential Geometric Approach." (1996).

Introduction to Econophysics and Possible Applications:

Econophysics is an interdisciplinary field that applies ideas and methods from statistical physics to economic and financial systems. Instead of focusing only on representative agents or equilibrium states, econophysics emphasizes probability, fluctuations, networks, and non-linear dynamics to explain real-world economic behaviour.

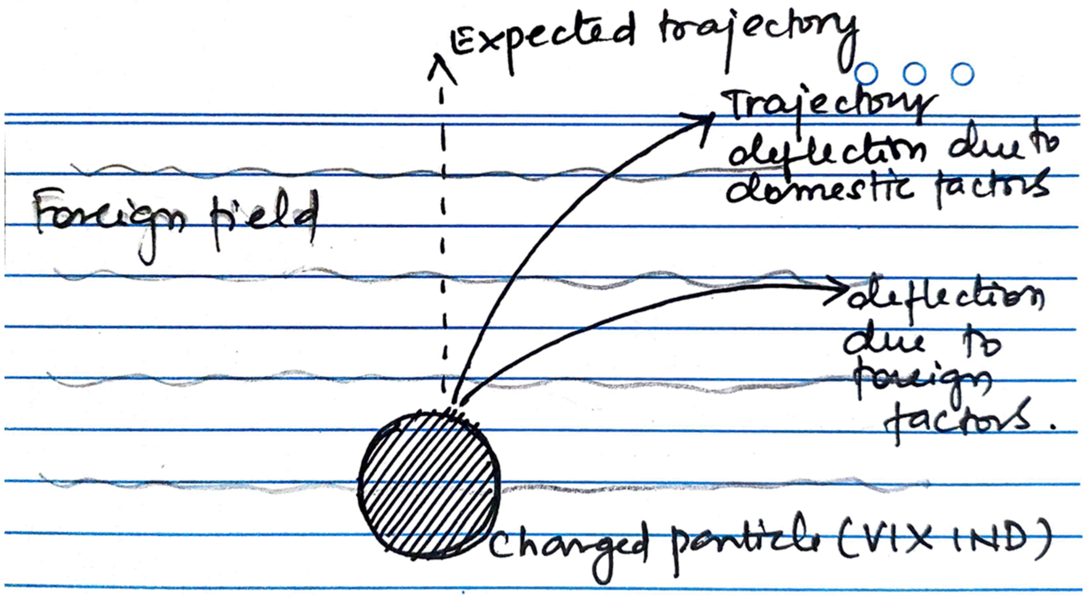
The similarity between economics and physics is not surprising. Both fields deal with systems composed of many interacting units - particles in physics and agents in economics - where individual behaviour is uncertain but aggregate patterns are often stable and predictable. Concepts such as random motion, diffusion, entropy, and phase transitions naturally map onto price movements, volatility, uncertainty, and market crashes. Historically, economics has already drawn heavily from physics, from Brownian motion in asset pricing to modern complexity and network models.

Differential geometry studies how systems move and interact on curved, interconnected spaces, where local changes are shaped by the overall structure. Gauge theory adds the idea that what we observe depends on the reference frame, while the core relationships stay unchanged. Intuitively, both help explain complex systems where interactions and relative changes matter more than absolute levels.

A simple example is a charged electron moving inside an electromagnetic field. The field changes the direction of the electron at every point. Instead of thinking of a “force” pushing it, differential geometry treats the field as something that changes the shape of the space the electron is moving through, so the electron follows a curved path that comes naturally from that shape.

The same idea can be applied to volatility transmission in finance. Each market’s volatility can be thought of as a surface that changes over time. When a foreign market affects a domestic market, it changes the shape of this surface, just as a field changes the space around the electron. The “imported” part of volatility is the part of the domestic surface that bends because of movements in another market.
Differential geometry gives a way to separate the part of volatility that comes from inside the domestic market from the part that comes from outside influences, by looking at how the shape of the volatility surface changes due to these external connections.

Below would be brief insight into understanding volatility transmission - overlapping the electron example and the possibility of understanding “import” of volatility through VIX_IND and VIX_US.



Establishing the behaviour of volatility as a system with curvature changes could potentially help us understand behaviours in more complex overlapping systems.

Curvature, in simple terms, means "how sharply something bends."

When the foreign market becomes volatile, the domestic volatility surface bends in response. Differential geometry provides a way to measure how much of this bending is due to foreign influence.

$$\underbrace{D_t \sigma_{IN_t}}_{\text{domestic residual of volatility}} = \underbrace{\nabla \sigma_{IN_t}}_{\substack{\text{domestic "field"} \\ (\sigma_t - \sigma_{t-1})_{IN} \\ \text{(gradient)}}} - \underbrace{\varphi_t G_{(t-1)} \sigma_{US(t-1)}}_{\text{foreign (US) "field" creating change in exp. curvature.}}$$

WHERE:

$$G_t = ER_t \times \left(\underbrace{e^{(\pi_{IN} - \pi_{US})}}_{\text{daily inflation adj. factor.}}\right)^{(1/n)} \cancel{\times} \cancel{\frac{6}{10}}.$$

In conclusion, calculating the imported component of domestic volatility using differential geometry is entirely possible, but it requires careful construction of the volatility “surface” shared by the domestic and foreign markets.
The key idea is that volatility does not move independently across markets; instead, the joint evolution of two markets forms a curved surface where movements in one direction affect the curvature in another. Differential geometry provides a structured way to measure this interaction by identifying how changes in foreign volatility alter the direction and curvature of domestic volatility through a connection term.

Once this connection is estimated from data, the imported component of volatility can be separated cleanly from the domestic component using a simple decomposition. Although this method is less common than standard econometric approaches, it is conceptually sound, mathematically consistent, and well-suited for capturing nonlinear and state-dependent spillovers that traditional models may struggle to represent.

Literature Review:

Empirical findings from both academic and policy circles reinforce that India's financial markets have become progressively more integrated into the world. The implication is that domestic volatility can be sparked by events far beyond India's borders.

This has spurred Indian policymakers to monitor global developments closely. Take for example how the RBI's Financial Stability Reports[5] include analyses of how external shocks might propagate to Indian markets. It also strengthens the case for building a composite index to quantify India's sensitivity to external volatility, as discussed next.

External organizations have also attempted to look into construction of composite indices for the specific case of Indian volatility as in the case of the 2015 'India - Selected Issues' report by IMF[6] - which looked into a dynamic multi-country GVAR framework based on an extended version of the model given by Cashin et al. (2012,2014).

Focusing on the literature of such attempts at quantification of volatility transmission leads us to many diverse cases of model methodologies, timelines and conclusions.

Some early studies, such as Chaudhuri and Koo[7](2001), highlighted that both domestic macroeconomic fundamentals and external factors, including U.S. stock market fluctuations and exchange rate volatility, significantly explain stock return volatility in Asian emerging markets, implying strong contagion and financial integration. Using a VAR-based dynamic framework, they found evidence of regional co-movement, particularly with Japan exerting dominant influence in Asia rather than the U.S.

Methodologically, Karolyi[8] (1995) advanced this field by applying a bivariate and multivariate GARCH model to Canada/USA equity data, enabling estimation of how innovations in one market's returns affect both mean and conditional volatility in another. This model formalized the feedback process of volatility spillovers and emphasized the role of cross-market conditional variance linkages in explaining return co-movements

It is also important to acknowledge the widely used seminal work by Baba, Engel, Kraft and Kroner that introduced the BEKK [9]multivariate GARCH model, which became one of the most important tools for studying how volatility moves between markets. Their model was designed to

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[5] RBI-FINANCIAL-STABILITY-REPORT-30-06-25.pdf

[6] https://www.imf.org/external/pubs/ft/scr/2015/cr1562.pdf#:~:text=construct%20a%20series%20of%20financial,IMF%E2%80%99s%20Coordinated%20Portfolio%20Investment%20Survey

[7] https://www.epw.in/journal/2001/40/special-articles/volatility-stock-returns.html

[8] https://ideas.repec.org/a/bla/jfinan/v51y1996i3p951-86.html

[9] https://www.estima.com/ecourse/samples/GARCHSampleChapter.pdf

Engle, Robert F., and Kenneth F. Kroner. "Multivariate Simultaneous Generalized ARCH." Econometric Theory, vol. 11, no. 1, 1995, pp. 122-150.

make sure that the estimated covariance matrix (which shows how market volatilities move together) is always positive and stable, something earlier models struggled with.

Chancharoenchai and Dibooglu(2005) used a trivariate and bivariate GARCH-M (BEKK parameterization) to model excess return volatility during the Asian Financial Crisis, confirming significant volatility transmission from U.S. and Japanese markets to regional markets and identifying asymmetric spillover effects under crisis conditions

In the domain of high-frequency data, Melvin and Peiers (2003) decomposed global foreign exchange volatility into regional "heat wave" (own-region persistence) and "meteor shower" (inter-regional spillover) effects. Their integrated volatility framework used realized volatility measures from intraday squared returns, revealing that local volatility persistence dominates interregional spillovers, though statistically significant cross-market transmission exists, particularly from Asian to Western sessions. This has further been compounded by work like that of John Beirne et al. (2015) on transmission from mature to emerging markets.[10]

These empirical advancements set the foundation for modern spillover measurement techniques like the Diebold–Yilmaz (2012) index.

The Diebold–Yilmaz spillover index[11] quantifies how volatility (or returns) in one market contributes to variability in others using a forecast error variance decomposition (FEVD) from a vector autoregression (VAR) model.

After estimating the VAR on a set of variables (such as returns, or realized volatilities etc.), the method decomposes each variable's forecast error variance into parts attributed to shocks from itself and from other variables. The share of variance in one market explained by shocks from another reflects the directional spillover, while the sum of all cross-market contributions represents the total spillover index.

By using generalized FEVD (which avoids dependence on variable ordering) and applying it in rolling or time-varying windows, the Diebold–Yilmaz framework captures both the magnitude and evolution of volatility transmission across markets over time.

Infact, the RBI uses a customized form of this methodology to construct indices for tracking time-varying volatility spillovers and interconnectedness across various financial markets (like the equity, currency, bond markets etc).

Other methodologies used in existing literature:

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[10] https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1113.pdf#:~:text=interdependencies%20in%20second%20moments%2C%20we,as%20a%20shift%20in%20the

[11] Diebold, Francis X., and Kamil Yilmaz. Measuring Financial Asset Return and Volatility Spillovers. NBER Working Paper No. 13811, National Bureau of Economic Research, 2008.

In this section, I cover a large set of existing volatility transmission models in a non-technical manner for ease of understanding and to provide a basis for the comparative work that is to follow.

Broadly speaking, much of the models have been formed through a quantitative financial frameworks, where “volatility” is modelled as a time-varying conditional second moment (conditional variance or volatility process).

Approaches fall roughly into:

- Parametric conditional variance models (ARCH/GARCH family)
- Stochastic volatility models (latent volatility driven by an unobserved process)
- Realized volatility / nonparametric HF-based measures and their time-series models (HAR, MIDAS)
- Multivariate models to capture co-movements and spillovers (BEKK, DCC, VAR/TVP-VAR, factor models)
- Spillover / connectedness metrics (As mentioned before - the DieboldYilmaz)

Each class has tradeoffs such as computational cost, interpretability, data needs, and ability to capture dynamics like leverage, fat tails, and frequency-dependent connectivity.

The tables as part of this report summarizes in approximate chronological order the advancement in academic research by tracking the evolution of models used to understand and calculate volatility transmission across markets.

Table 1: Chronological record of models for evaluation of volatility transmission across markets.

Table 2: Summarized record of seminal papers on the subject of calculation of such volatility spillovers.

Significance of Econophysics outlook: Differential Geometry Methodology.